1 **B**

2 **B** We first must find $\cos A$ and $\cos B$. Since both angle are acute, we know that

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13},$$
$$\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\left(\frac{8}{17}\right)^2} = \frac{15}{17}.$$

Therefore,

$$\begin{aligned}\sin(A-B) &= \sin A\cos B - \cos A\sin B\\ &= \frac{5}{13}\frac{15}{17} - \frac{12}{13}\frac{8}{17}\\ &= -\frac{21}{221}.\end{aligned}$$

3 **E**

4 **E** We first must find $\cos A$ and $\cos B$. Since both angle are acute, we know that

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}, \cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\left(\frac{8}{17}\right)^2} = \frac{15}{17}.$$

Therefore,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12},$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}.$$

Therefore,

$$\begin{aligned}\tan(A+B) &= \frac{\tan A + \tan B}{1-\tan A\tan B}\\ &= \frac{\frac{5}{12}+\frac{8}{15}}{1-\frac{5}{12}\frac{8}{15}}\\ &= -\frac{171}{140}.\end{aligned}$$

5 **D** Rearranging the equation, gives

$$\sin\left(x-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

We know that solution must be in the first and second quadrants. The principle angle is $\frac{\pi}{3}$. Therefore,

$$\begin{aligned}x-\frac{\pi}{6} &= \frac{\pi}{3}, \frac{2\pi}{3}\\ &= \frac{2\pi}{6}, \frac{4\pi}{6}\\ x &= \frac{\pi}{2}, \frac{5\pi}{6}\end{aligned}$$

6 **D** If $\cos\theta = c$ and θ is acute then

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-c^2}.$$

Therefore,

$$\begin{aligned}\cot\theta &= \frac{\cos\theta}{\sin\theta}\\ &= \frac{c}{\sqrt{1-c^2}}.\end{aligned}$$

7 **C** As the period is 120°, we know that the coefficient of a must be

$$\frac{360^\circ}{120^\circ} = 3.$$

As the range is $[-4, 2]$, we know that the vertical translation is -1. This leaves only items B and C. Now let $a = 0$ so to see which of these gives the correct value of y. We have,

$$y = 3\cos(0) - 1 = 3 - 1 = 2,$$

$$y = -3\sin(0) - 1 = 0 - 1 = -1.$$

8 **A** The each graph has the same amplitude. Item A has period 2π. Item B has period

$$\frac{2}{\pi} \div \frac{1}{2} = 4\pi.$$

Therefore the second graph has twice the period of the first.

9 **D** We have,

$$\begin{aligned}\cos A\cos B - \sin A\sin B &= \cos(A+B)\\ &= \cos\frac{\pi}{2}\\ &= 0.\end{aligned}$$

10 **E** We first must find $\cos A$. Since A is obtuse, we know that

$$\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\left(\frac{\sqrt{5}}{3}\right)^2} = -\frac{2}{3}.$$

Therefore, $\sin(2A) = 2\sin A\cos A$

$$\begin{aligned}&= -2\times\frac{\sqrt{5}}{3}\times\frac{2}{3}\\ &= -\frac{4\sqrt{5}}{9}\end{aligned}$$

11 **A** Note that the graph does not show one full cycle of the function. The biggest clue, is the fact that $y = 0$ when $x = \frac{\pi}{12}$. Only items A and E have this property. For item E, when $x = 0$ we have $y = -\sin\left(-\frac{\pi}{6}\right) > 0$. This does not agree with the figure shown, leaving only item A.

12 **A** The function has period $\frac{180^\circ}{3} = 60^\circ$. This leaves items A,B and C. One cannot speak of the amplitude of the tangent function. This leaves only items A and C. Since $h(x) = -f(x)$, the graph of f will be a reflection of the graph of h in the x-axis. The graph is not a reflection of g as g does not have the appropriate dilation factors. This leaves item A.

13 **E** If $\cos\theta = c$ and θ is acute then

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-c^2}.$$

Therefore, $\sin(2\theta) = 2\sin\theta\cos\theta$

$$= 2c\sqrt{1-c^2}.$$

14 **C** This is most efficiently solved using your calculator, giving, 41.50° and 244.67°.

15 **C** $$\begin{aligned} 8\sin\theta\cos^3\theta - 8\sin^3\theta\cos\theta &= 8\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta) \\ &= 4\sin(2\theta)\cos(2\theta) \\ &= 2\sin(4\theta). \end{aligned}$$

16 **C** The graph of $y = \tan x$ has been

- dilated from the y-axis by a factor of 2
- translated $\frac{\pi}{2}$

There is no vertical translation. Therefore the rule is of the form

$$y = a\tan\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right).$$

This leaves only item C. One can confirm this by checking that $y = -1$ when $x = 0$.

17 **B** We have,

$$\begin{aligned} z &= vw \\ &= 4\text{cis}(-0.3\pi) \times 5\text{cis}(-0.6\pi) \\ &= 20\text{cis}(-0.3\pi + (-0.6\pi)) \\ &= 20\text{cis}(-0.9\pi) \end{aligned}$$

so that $\text{Arg}z = -0.9\pi$.

18 **D** $$\begin{aligned} 2\text{cis}\left(\frac{2\pi}{3}\right) &= 2(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)) \\ &= 2(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) \\ &= -1 + \sqrt{3}i \end{aligned}$$

19 **E** This complex number is in the third quadrant. Moreover, since

$$\tan\theta = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}},$$

it is clear that $\theta = -\frac{5\pi}{6}$.

20 **A** The imaginary part of a complex number is the coefficient of i. In this case, the coefficient is -3.

21 **C** $$\begin{aligned} uv &= 3\text{cis}\left(\frac{\pi}{2}\right) \times 5\text{cis}\left(\frac{2\pi}{3}\right) \\ &= 15\text{cis}\left(\frac{\pi}{2} + \frac{2\pi}{3}\right) \\ &= 15\text{cis}\left(\frac{3\pi}{6} + \frac{4\pi}{6}\right) \\ &= 15\text{cis}\left(\frac{7\pi}{6}\right) \\ &= 15\text{cis}\left(\frac{7\pi}{6} - 2\pi\right) \\ &= 15\text{cis}\left(-\frac{5\pi}{6}\right) \end{aligned}$$

22 **C** The modulus is given by

$$|12-5i| = \sqrt{12^2+(-5)^2} = 13.$$

23 A

- z^2 need not be real. For example, $(1+i)^2 = 2i$ is not a real number.
- Since $z\overline{z} = |z|^2$, this will always be a real number.
- Since $z^{-1}z = 1$, this will always be a real number.
- $\text{Im}z$ is the coefficient of i, and so will be real.
- Since $z+\overline{z} = 2\text{Re}z$, this will be real.

24 C $\overline{z} = -14+7i.$

25 E Factorising the expression gives,

$$\begin{aligned}3z^2+9 &= 3(z^2+3)\\ &= 3(z^2-(\sqrt{3}i)^2)\\ &= 3(z-\sqrt{3}i)(z+\sqrt{3}i).\end{aligned}$$

26 C Expanding the brackets gives,

$$\begin{aligned}(1+2i)^2 &= 1+4i+(2i)^2\\ &= 1+4i-4\\ &= -3+4i.\end{aligned}$$

27 A As the graph has asymptote at $x=\pm 2$, the denominator of the function must be equal to zero when $x=\pm 2$. This leaves only items A and B. For item B, if $x=0$ then $y=-\frac{1}{4}$. This does not agree with the given graph, leaving only item A.

28 B The graph of $y = a\sin(x)+b$ needs to have two x-intercepts. This will happen provided that the amplitude a exceeds the vertical translation term b. that is, $a>b$.

29 C

30 B Since the distance from fixed point A to point $P(x,y)$ is a constant, the set of points must be a circle.

31 B Since $AP = BP$ for each point $P(x,y)$, the line $y=x+1$ is the perpendicular bisector of line AB. The line the perpendicular bisector of points $A(0,0)$ and $B(-1,1)$, but none of the other pairs.

32 C

- This is false. The axis of symmetry will be $x=0$.
- The parabola will not go through the origin as the distance from $(0,0)$ to $F(0,2)$ is 2 while the distance from $(0,0)$ to $y=-4$ is 4.
- This is true. The distance from $F(0,2)$ to $(0,-1)$ is 3 is equal to the distance from $y=-4$ to $(0,-1)$.
- This is false. The distance from $F(0,2)$ to $(1,2)$ is not equal to the distance from $y=-4$ to $(1,2)$.
- This cannot be the equation of the parabola, as the parabola must go through the point $(0,-1)$ and so has a y-intercept of -1.

33 B The hyperbola has x-intercpets at $x=\pm 1$. The ellipse will have x-intercepts at $x=\pm a$. Therefore, to have four points of intersection we require that $a>1$.

34 D To find the centre of the hyperbola, we can find the point of intersection of the asymptotes. To find this, we solve, $\begin{aligned}2x+1 &= -2x+1\\ 4x &= 0\end{aligned}$ Therefore, $y=1$ and the centre is $(0,1)$. This leaves items A,B and D. The graph

$x = 0.$

has no x-axis intercept. Therefore we can exclude items A and B. This leaves item D.

35 **A** Since $x = 1 + t$, we know that $t = x - 1$. Substituting $t = x - 1$ into the second equation gives,

$$\begin{aligned} y &= \frac{1-t}{1+t} \\ &= \frac{1-(x-1)}{1+(x-1)} \\ &= \frac{2-x}{x} \\ &= \frac{2}{x} - 1. \end{aligned}$$

36 **E** We can write this equation as

$$\left(\frac{x-1}{2}\right)^2 + \left(\frac{y+1}{3}\right)^2 = 1$$

So we can set

$$\begin{aligned} \cos t &= \frac{x-1}{2}, \\ \sin t &= \frac{y+1}{3}, \end{aligned}$$

so that

$$\begin{aligned} x &= 2\cos(t) + 1, \\ y &= 3\sin(t) - 1. \end{aligned}$$

37 **D** Notice that in each of the items the x-coordinate is 5. So solving $5 = 2t - 3$ for t gives $t = 4$. Now let $t = 4$ in the second equation to give

$$y = 4^2 - 3 \times 4 = 16 - 12 = 4.$$

Therefore the coordinates are $(5, 4)$.

38 **A** Since $y = r\sin\theta$ and $x = r\cos\theta$, we obtain,

$$\begin{aligned} r^2\cos^2\theta &= r\sin\theta \\ r &= \frac{\sin\theta}{\cos^2\theta} \\ &= \sec\theta\tan\theta. \end{aligned}$$